ISMIR 2004, Barcelona, Spain
5th International Conference on Music Information Retrieval
Audiovisual Institute, Universitat Pompeu Fabra
October 10, 2004, 1600h-1900h

Musical Knowledge • Computational Models • Retrieval

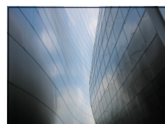
Elaine Chew
University of Southern California Viterbi School of Engineering
Epstein Department of Industrial and Systems Engineering
Integrated Media Systems Center
www-rcf.usc.edu/~echew
echew@usc.edu

Goals for the afternoon

- Music Perception and Cognition
  - What can we hear?
  - How can we describe it?
- Modeling Musical Intelligence
  - Focus on pitch structures
  - [1] Modeling tonality
  - Computational music cognition
    - [2] Key Finding
    - [3] Segmentation
    - [4] Pitch Spelling

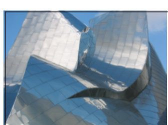
Implications for Retrieval

- Exploit music theoretic knowledge
  - avoid re-inventing the wheel
  - understand the subject at hand
- Create content-based description
  - context, boundaries
  - summarization, indexing
  - similarity assessment
- Build user-centered systems
  - perceptually and cognitively-inspired descriptors
  - human-level apprehension of music



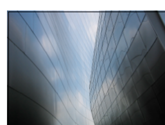
What can we hear?

- stable pitches
  - LISTEN *Simple Gifts* from Copland's *Appalachian Spring*
  - LISTEN Schubert *Vier Impromptus* No.3 D 935 theme
  - LISTEN suggestions welcome
- ordered sets
  - LISTEN Twinkle Twinkle Little Star
  - LISTEN Joy to the World
  - LISTEN starting from "doh", octave equivalence



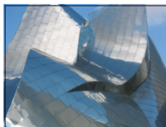
What can we hear?

- context
  - LISTEN Schubert *Vier Impromptus* No.3 D 935 spliced
  - LISTEN Mozart *Rondo* K.511
  - LISTEN random example
- context change
  - LISTEN Schubert *Vier Impromptus* No.3 D 935, 2nd half
  - LISTEN Mozart *Rondo* K.511, continued
  - LISTEN Bach Minuet in G



What can we hear?

- similarity
  - LISTEN Schubert *Vier Impromptus* No.3 D 935 theme
  - LISTEN Schubert *Vier Impromptus* No.3 D 935 var x
  - LISTEN Mozart *Var on "Ah, vous dirais-je, Maman"* theme
  - LISTEN Mozart *Var on "Ah, vous dirais-je, Maman"* var x
  - LISTEN Beethoven *Piano Sonata* Op.79 mvt 3
  - LISTEN Beethoven *Piano Sonata* Op.109 mvt 1



How can we describe it?

- scales of description
  - local, global
  - note, cluster, context
- frame of reference

- What is a pitch?
  - A, B, C, #, b
  - pitch class notation
- What is an interval?
  - major/minor
  - augmented/diminished
- What is a chord / triad?
  - I, IV, V
  - ii, vi, iii
- What is a key?
  - “doh” (tonic)

Modeling Tonality: from Experience to Description

- A walk through *some* history of tonality models
  - Shephard (psychology) □ Krumhansl □ Lerdahl
  - Euler (mathematics) □ Riemann □ Lewin □ Cohn
  - Longuet-Higgins (cognitive science) □ Steedman
  - → Chew
- The Spiral Array

Modeling Tonality: Roger N. Shephard

Photo from voteview.uh.edu

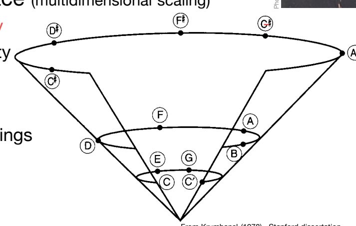
- Mental models (1982)
- Multi-dimensional scaling

“the cognitive representation of musical pitch must have properties of great regularity, symmetry, and transformational invariance.”

Modeling Tonality: Carol Krumhansl

Photo from Cornell Chronicle, Feb 2000

- The Basic Space (multidimensional scaling)
  - pitch proximity
  - chord proximity
  - key proximity
- Application
  - probe tone ratings
  - Key-finding



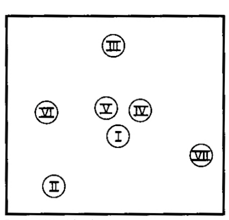
From Krumhansl (1978). Stanford dissertation.

LISTEN

Modeling Tonality: Carol Krumhansl

Photo from Cornell Chronicle, Feb 2000

- The Basic Space (multidimensional scaling)
  - pitch proximity
  - chord proximity
  - key proximity
- Application
  - probe tone ratings
  - Key-finding



From Lerdahl book (2001).

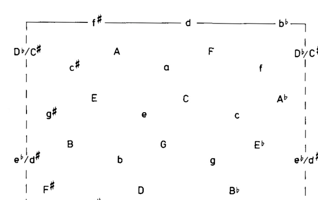
LISTEN

Mozart *Var on “Ah, vous dirais-je, Maman”* theme

Modeling Tonality: Carol Krumhansl

Photo from Cornell Chronicle, Feb 2000

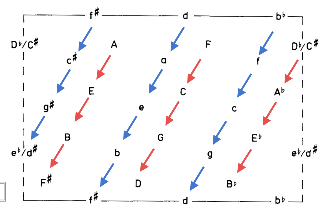
- The Basic Space (multidimensional scaling)
  - pitch proximity
  - chord proximity
  - key proximity
- Application
  - probe tone ratings
  - Key-finding



From Krumhansl (1990) p.46

Modeling Tonality: Carol Krumhansl

- The Basic Space (multidimensional scaling)
  - pitch proximity
  - chord proximity
  - key proximity
    - stepping by fifths
    - relative major/minor
    - parallel major/minor
- Application
  - probe tone ratings
  - Key-finding LISTEN

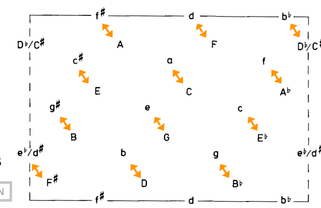


Bach's Minuet in G

From Krumhansl (1990) p.46

Modeling Tonality: Carol Krumhansl

- The Basic Space (multidimensional scaling)
  - pitch proximity
  - chord proximity
  - key proximity
    - stepping by fifths
    - relative major/minor
    - parallel major/minor
- Application
  - probe tone ratings
  - Key-finding LISTEN

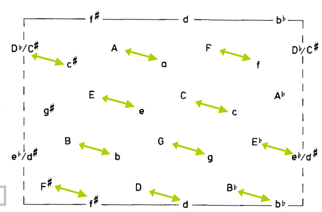


Beethoven Variations on "God Save the King" var V

From Krumhansl (1990) p.46

Modeling Tonality: Carol Krumhansl

- The Basic Space (multidimensional scaling)
  - pitch proximity
  - chord proximity
  - key proximity
    - stepping by fifths
    - relative major/minor
    - parallel major/minor
- Application
  - probe tone ratings
  - Key-finding LISTEN

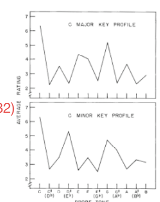


Mozart Var on "Ah, vous dirais-je, Maman" var 8

From Krumhansl (1990) p.46

Modeling Tonality: Carol Krumhansl

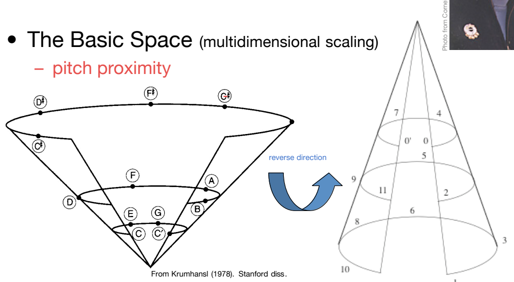
- Probe tone studies (multidimensional scaling)
  - pitch proximity
  - chord proximity
  - key proximity
- Probe tone profiles
  - probe tone ratings (Krumhansl & Kessler, 1982)
  - Key-finding (Krumhansl & Schuckler, 1990)



From Krumhansl (1990) p.31

Modeling Tonality: Carol Krumhansl

- The Basic Space (multidimensional scaling)
  - pitch proximity



From Krumhansl (1978). Stanford diss.

Modeling Tonality: Fred Lerdahl

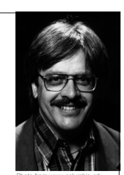
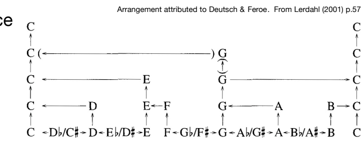
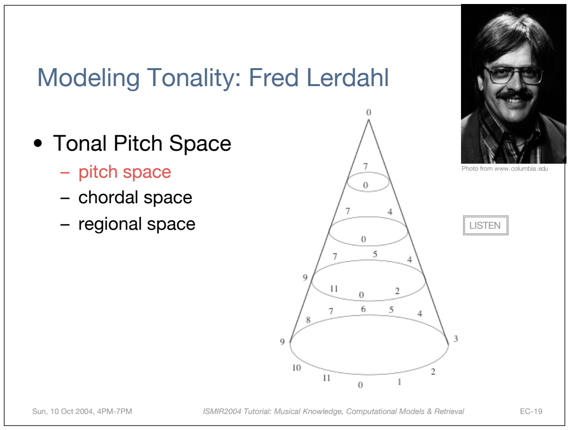


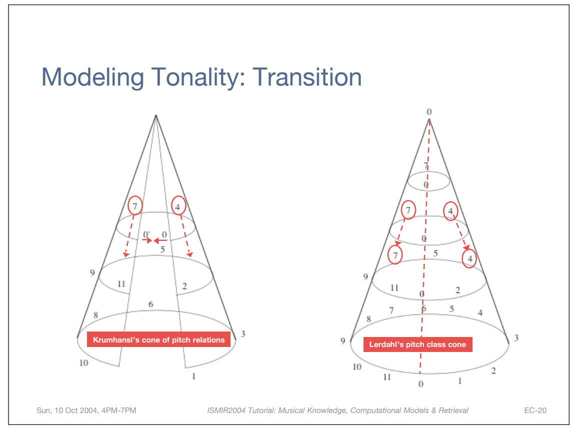
Photo from www.columbia.edu

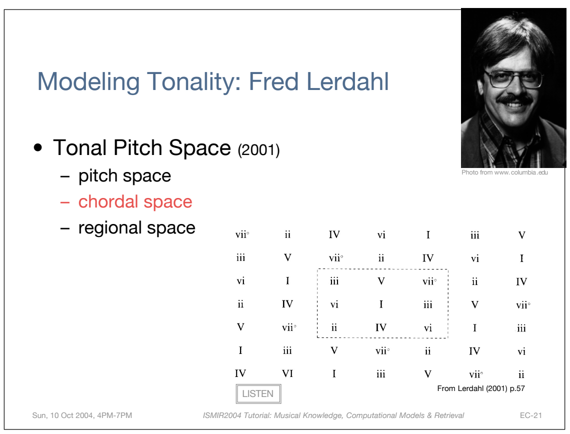
- Tonal Pitch Space
  - pitch space
  - chordal space
  - regional space

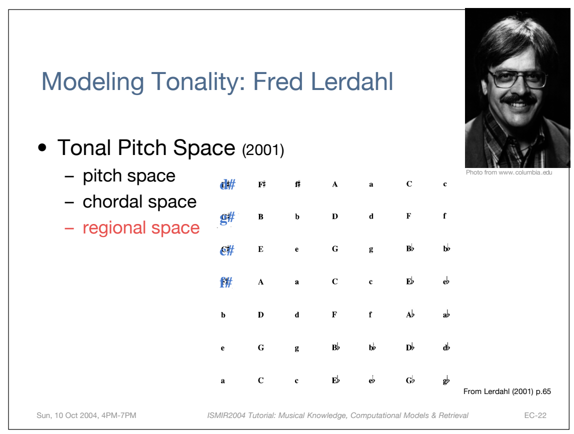
Arrangement attributed to Deutsch & Feroe. From Lerdahl (2001) p.57

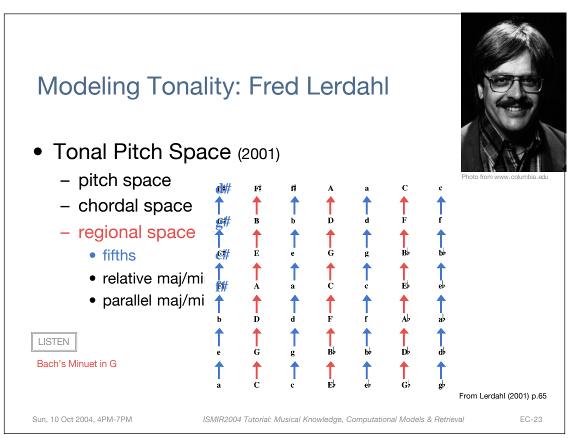


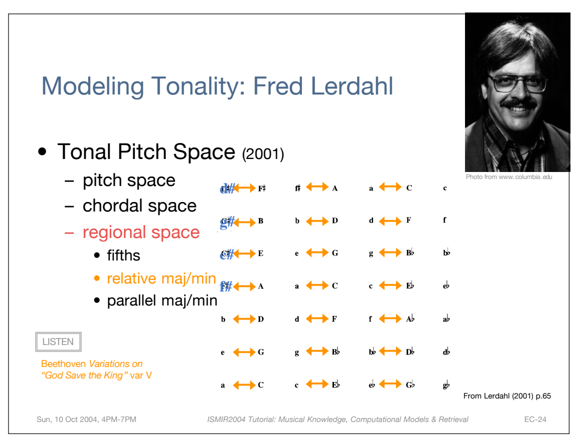

Modeling Tonality: Fred Lerdahl
• Tonal Pitch Space
– pitch space
– chordal space
– regional space
Photo from www.columbia.edu
LISTEN
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-19

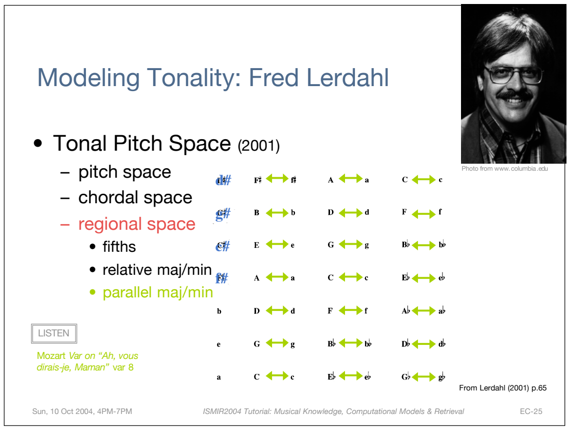

Modeling Tonality: Transition
Krumhansl's cone of pitch relations
Lerdahl's pitch class cone
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-20

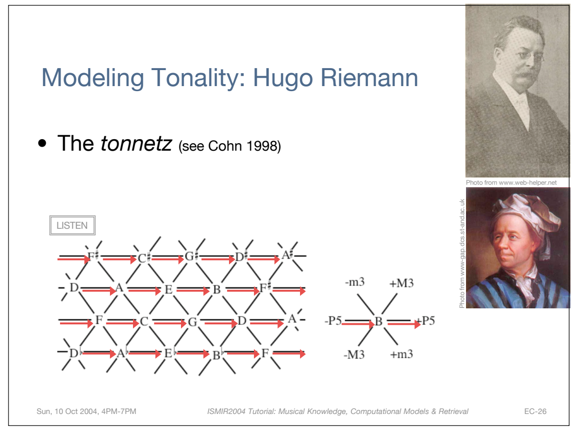

Modeling Tonality: Fred Lerdahl
• Tonal Pitch Space (2001)
– pitch space
– chordal space
– regional space
Photo from www.columbia.edu
LISTEN
From Lerdahl (2001) p.57
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-21

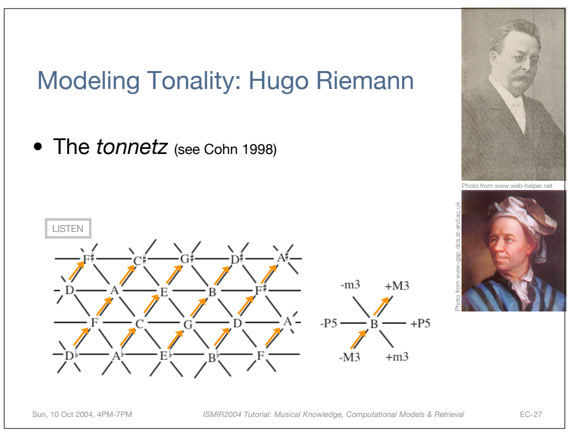

Modeling Tonality: Fred Lerdahl
• Tonal Pitch Space (2001)
– pitch space
– chordal space
– regional space
Photo from www.columbia.edu
From Lerdahl (2001) p.65
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-22

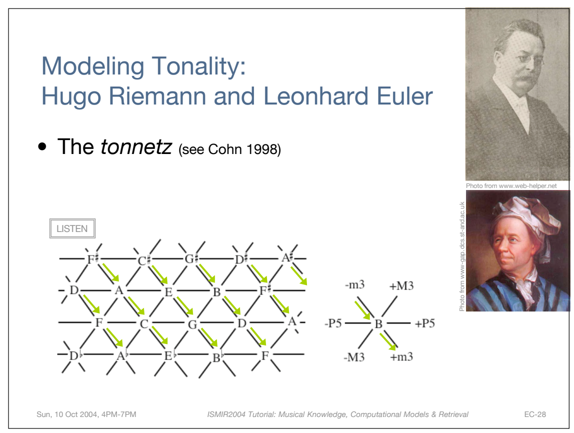

Modeling Tonality: Fred Lerdahl
• Tonal Pitch Space (2001)
– pitch space
– chordal space
– regional space
• fifths
• relative maj/mi
• parallel maj/mi
Photo from www.columbia.edu
LISTEN
Bach's Minuet in G
From Lerdahl (2001) p.65
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-23

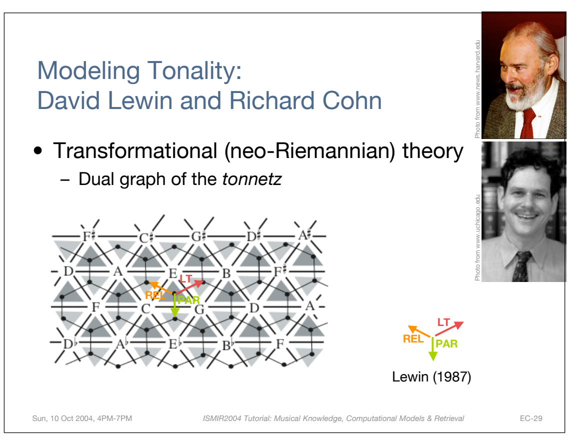

Modeling Tonality: Fred Lerdahl
• Tonal Pitch Space (2001)
– pitch space
– chordal space
– regional space
• fifths
• relative maj/min
• parallel maj/min
Photo from www.columbia.edu
LISTEN
Beethoven Variations on "God Save the King" var V
From Lerdahl (2001) p.65
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-24

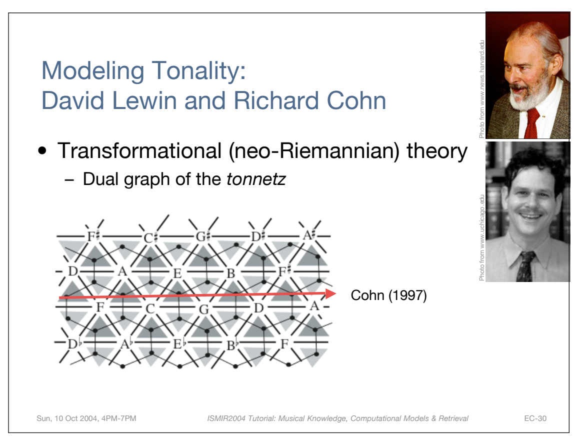

Modeling Tonality: Fred Lerdahl
Tonal Pitch Space (2001)
pitch space
chordal space
regional space
fifths
relative maj/min
parallel maj/min
LISTEN
Mozart Var on "Ah, vous dirais-je, Maman" var 8
From Lerdahl (2001) p.65
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-25


Modeling Tonality: Hugo Riemann
The tonnetz (see Cohn 1998)
LISTEN
-m3
+M3
-P5
+P5
-M3
+m3
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-26

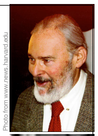

Modeling Tonality: Hugo Riemann
The tonnetz (see Cohn 1998)
LISTEN
-m3
+M3
-P5
+P5
-M3
+m3
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-27


Modeling Tonality:
Hugo Riemann and Leonhard Euler
The tonnetz (see Cohn 1998)
LISTEN
-m3
+M3
-P5
+P5
-M3
+m3
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-28

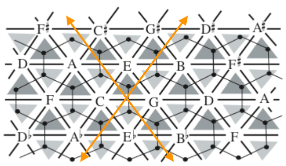

Modeling Tonality:
David Lewin and Richard Cohn
Transformational (neo-Riemannian) theory
Dual graph of the tonnetz
LT
REL
PAR
Lewin (1987)
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-29


Modeling Tonality:
David Lewin and Richard Cohn
Transformational (neo-Riemannian) theory
Dual graph of the tonnetz
Cohn (1997)
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-30

Modeling Tonality: David Lewin and Richard Cohn



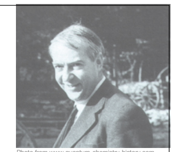

- Transformational (neo-Riemannian) theory
  - Dual graph of the *tonnetz*



Cohn (1997)

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-31

Modeling Tonality: Hugh Christopher Longuet-Higgins

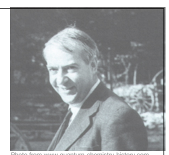


- Harmonic Network (1962a, 1962b)

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-32

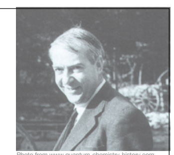
Modeling Tonality: Hugh Christopher Longuet-Higgins and Mark Steedman (1971)




E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-33

Modeling Tonality: Hugh Christopher Longuet-Higgins and Mark Steedman (1971)



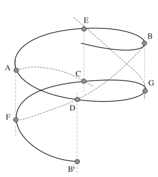

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-34

Modeling Tonality: Transition

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Longuet-Higgins' harmonic network

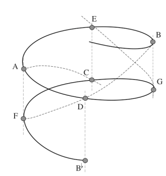


Chew's pc spiral in the spiral array

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-35

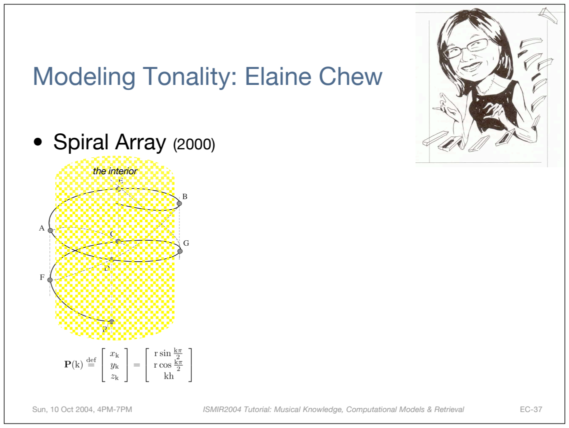
Modeling Tonality: Elaine Chew

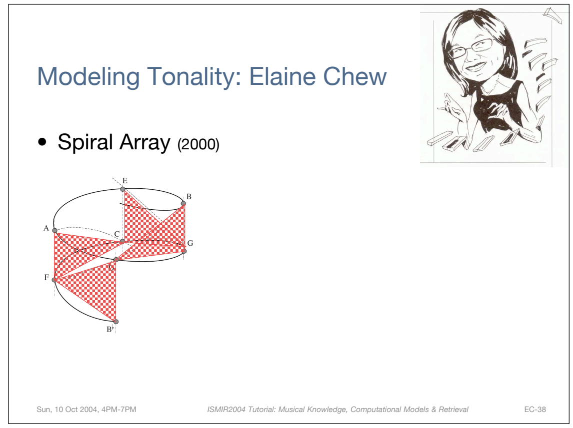
- Spiral Array (2000)

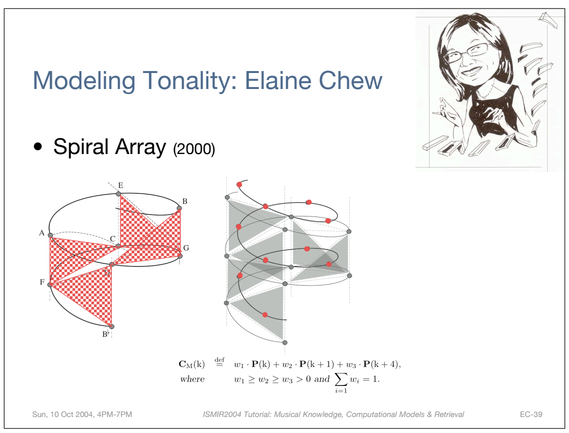


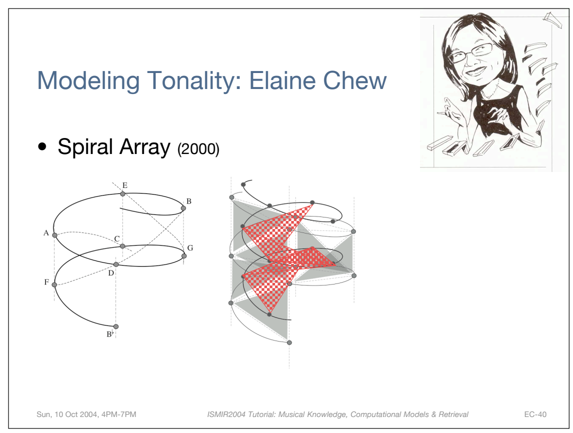
$$\mathbf{P}(k) \stackrel{\text{def}}{=} \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} = \begin{bmatrix} r \sin \frac{k\pi}{2} \\ r \cos \frac{k\pi}{2} \\ kh \end{bmatrix}$$

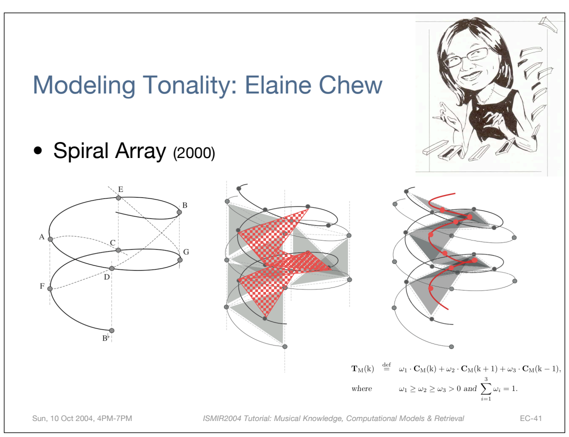
Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-36

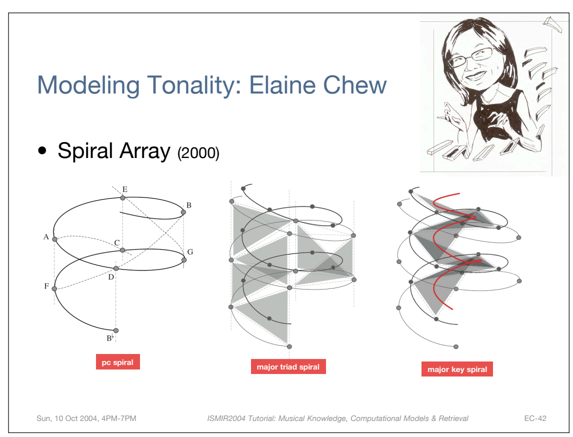

Modeling Tonality: Elaine Chew
• Spiral Array (2000)
the interior
E
B
A
C
G
F
B♭
P(k) def= [x_k y_k z_k] = [r sin kπ/2, r cos kπ/2, kh]
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-37

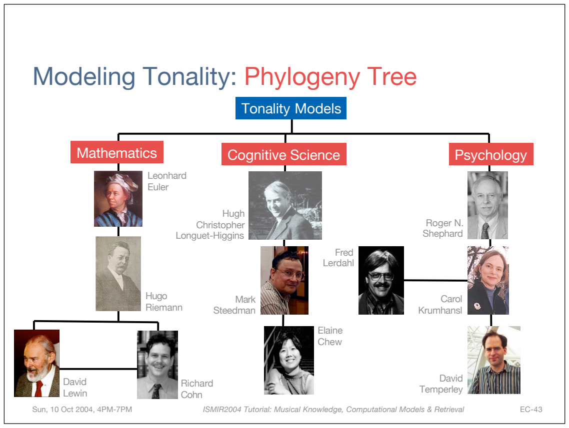

Modeling Tonality: Elaine Chew
• Spiral Array (2000)
E
B
A
C
G
F
B♭
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-38

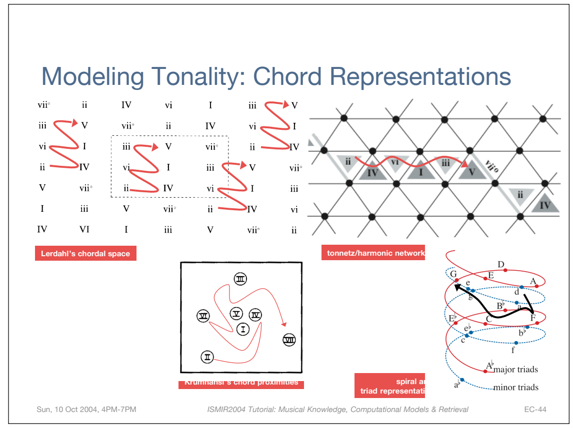

Modeling Tonality: Elaine Chew
• Spiral Array (2000)
E
B
A
C
G
F
B♭
C_M(k) def= w_1 · P(k) + w_2 · P(k+1) + w_3 · P(k+4),
where w_1 ≥ w_2 ≥ w_3 > 0 and Σ_{i=1}^{3} w_i = 1.
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-39

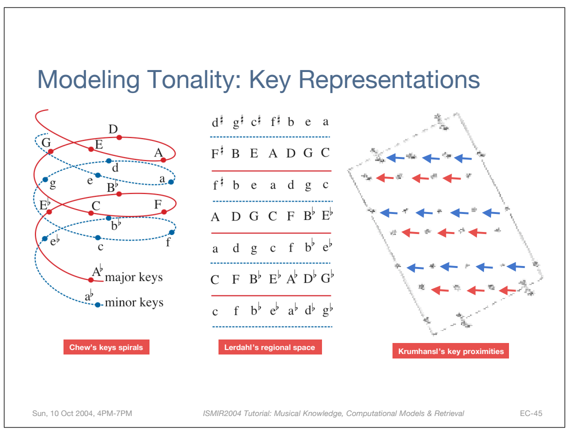

Modeling Tonality: Elaine Chew
• Spiral Array (2000)
E
B
A
C
G
D
F
B♭
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-40

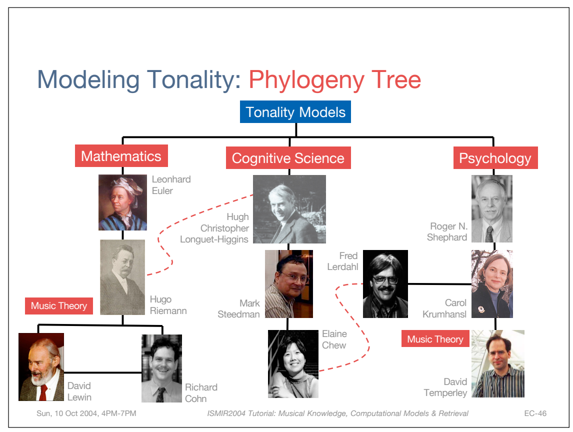

Modeling Tonality: Elaine Chew
• Spiral Array (2000)
E
B
A
C
G
D
F
B♭
T_M(k) def= w_1 · C_M(k) + w_2 · C_M(k+1) + w_3 · C_M(k−1),
where w_1 ≥ w_2 ≥ w_3 > 0 and Σ_{i=1}^{3} w_i = 1.
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-41

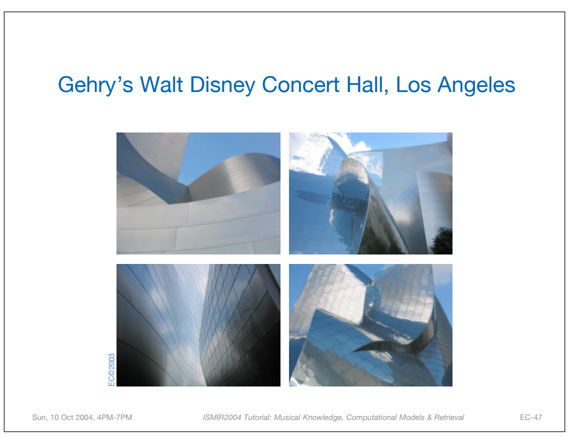

Modeling Tonality: Elaine Chew
• Spiral Array (2000)
E
B
A
C
G
D
F
B♭
pc spiral
major triad spiral
major key spiral
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-42


Modeling Tonality: Phylogeny Tree
Tonality Models
Mathematics
Cognitive Science
Psychology
Leonhard Euler
Hugh Christopher Longuet-Higgins
Roger N. Shephard
Fred Lerdahl
Hugo Riemann
Mark Steedman
Carol Krumhansl
Elaine Chew
David Lewin
Richard Cohn
David Temperley
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-43


Modeling Tonality: Chord Representations
Lerdahl's chordal space
tonnetz/harmonic network
major triads
minor triads
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-44


Modeling Tonality: Key Representations
major keys
minor keys
Chew's keys spirals
Lerdahl's regional space
Krumhansl's key proximities
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-45


Modeling Tonality: Phylogeny Tree
Tonality Models
Mathematics
Cognitive Science
Psychology
Leonhard Euler
Hugh Christopher Longuet-Higgins
Roger N. Shephard
Fred Lerdahl
Music Theory
Hugo Riemann
Mark Steedman
Carol Krumhansl
Elaine Chew
Music Theory
David Lewin
Richard Cohn
David Temperley
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-46


Gehry's Walt Disney Concert Hall, Los Angeles
EC©2003
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-47

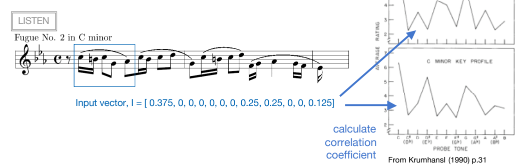

Goals for the afternoon
• Music Perception and Cognition
– What can we hear?
– How can we describe it?
• Modeling Musical Intelligence
– Focus on pitch structures
– [1] Modeling tonality
– Computational music cognition
• [2] Key Finding
• [3] Segmentation
• [4] Pitch Spelling
EC©2004
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-48

Key-Finding

- Krumhansl & Schmuckler (1990)
- Longuet-Higgins & Steedman (1971)
- Chew (2001)

Key-Finding: Krumhansl & Schmuckler

- Probe tone profiles
  - probe tone ratings (Krumhansl & Kessler, 1982)
  - Key-finding (Krumhansl & Schuckler, 1990)



From Krumhansl (1990) p.31

Key-Finding: Longuet-Higgins & Steedman

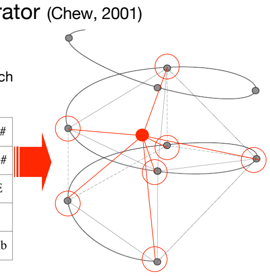
- Shape-matching (Longuet-Higgins & Steedman, 1971)
  - successively eliminate options
  - tonic-dominant rule

E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab

Key-Finding: Chew

- Center of Effect Generator (Chew, 2001)
  - Clustering of pitches in a key
  - generate center of effect
  - perform nearest neighbor search for closest key

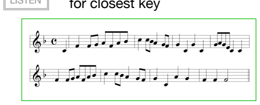
E	B	F#	C#	G#	D#	A#	E#	B#
C	G	D	A	E	B	F#	C#	G#
Ab	Eb	Bb	F	C	G	D	A	E
Fb	Cb	Gb	Db	Ab	Eb	Bb	F	C
Dbb	Abb	Ebb	Bbb	Fb	Cb	Gb	Db	Ab



Key-Finding: Chew

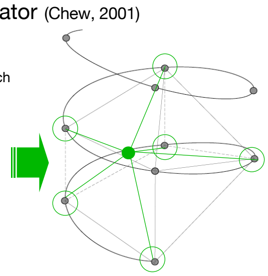
- Center of Effect Generator (Chew, 2001)
  - clustering of pitches in a key
  - generate center of effect
  - perform nearest neighbor search for closest key

LISTEN



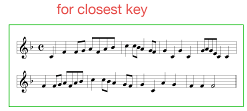
monophonic example

$$\mathbf{c}_{a,b} \stackrel{\text{def}}{=} \sum_{i=a}^{b}\sum_{j=1}^{n_i} \frac{d_{ij}}{D_{a,b}} \cdot \mathbf{p}_{ij}, \quad \text{where } D_{a,b} = \sum_{i=a}^{b}\sum_{j=1}^{n_i} d_{ij}.$$



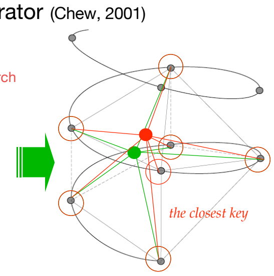
Key-Finding: Chew

- Center of Effect Generator (Chew, 2001)
  - clustering of pitches in a key
  - generate center of effect
  - perform nearest neighbor search for closest key



monophonic example

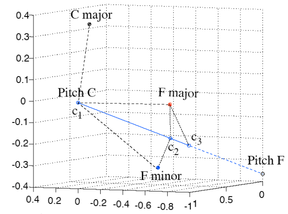
$$\mathbf{c}_{a,b} \stackrel{\text{def}}{=} \sum_{i=a}^{b}\sum_{j=1}^{n_i} \frac{d_{ij}}{D_{a,b}} \cdot \mathbf{p}_{ij}, \quad \text{where } D_{a,b} = \sum_{i=a}^{b}\sum_{j=1}^{n_i} d_{ij}.$$



Key-Finding: Chew

- Center of Effect Generator (Chew, 2001)

Simple Gifts from Copland's *Appalachian Spring*



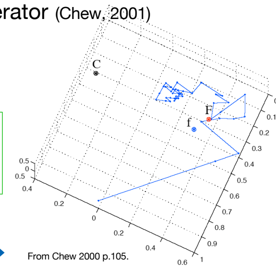
From Chew 2000 p.104.

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-55

Key-Finding: Chew

- Center of Effect Generator (Chew, 2001)

Simple Gifts from Copland's *Appalachian Spring*



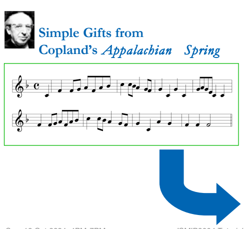
From Chew 2000 p.105.

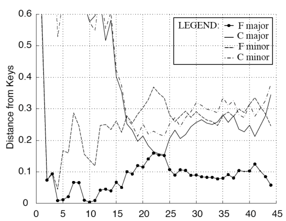
Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-56

Key-Finding: Chew

- Center of Effect Generator (Chew, 2001)

Simple Gifts from Copland's *Appalachian Spring*





From Chew 2000 p.106.

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-57

Key-Finding: Comparisons

J.S. Bach's *Well-Tempered Clavier* Bk 1

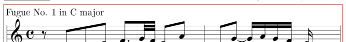
	Book 1 Fugue subjects	Steps to key		
		CEG[a]	PTPM	SMA
1	C major	2	2	16†[b]
2	C minor	5	5	5
3	C♯ major	6	7	16
4	C♯ minor	3	3	4
5	D major	2	2	15†
6	D minor	3	3	8
7	E♭ major	2	6	11†
8	D♯ minor	2	6	12†
9	E major	14	12†	11
10	E minor	3	2	7†
11	F major	4	10	6
12	F minor	3	15	4†
13	F♯ major	3	2	8
14	F♯ minor	7	18	5†
15	G major	2	2	15
16	G minor	3	3	4
17	A♭ major	3	2	7†
18	G♯ minor	5	5	5
19	A major	2	4	7
20	A minor	5	5	5
21	B♭ major	4	4	14
22	B♭ minor	2	3	6†
23	B major	2	11	11
24	B minor	3	3	7
	Average	[illegible]	5.25 (4.79)	8.71 (8.21)

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-58

Key-Finding: Comparisons

LISTEN

Fugue No. 1 in C major



J.S. Bach's *Well-Tempered Clavier* Bk 1

	Book 1 Fugue subjects	Steps to key		
		CEG[a]	PTPM	SMA
1	C major	2	2	16†[b]
2	C minor	5	5	5
3	C♯ major	6	7	16
4	C♯ minor	3	3	4
5	D major	2	2	15†
6	D minor	3	3	8
		2	6	11†
		2	6	12†
		14	12†	11
		3	2	7†
11	F major	4	10	6
12	F minor	3	15	4†
13	F♯ major	3	2	8
14	F♯ minor	7	18	5†
15	G major	2	2	15
16	G minor	3	3	4
17	A♭ major	3	2	7†
18	G♯ minor	5	5	5
19	A major	2	4	7
20	A minor	5	5	5
21	B♭ major	4	4	14
22	B♭ minor	2	3	6†
23	B major	2	11	11
24	B minor	3	3	7
	Average	[illegible]	5.25 (4.79)	8.71 (8.21)

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-59

Key-Finding: Comparisons

LISTEN

Fugue No. 3 in C♯ major

J.S. Bach's *Well-Tempered Clavier* Bk 1

	Book 1 Fugue subjects	Steps to key		
		CEG[a]	PTPM	SMA
1	C major	2	2	16†[b]
2	C minor	5	5	5
3	C♯ major	6	7	16
4	C♯ minor	3	3	4
5	D major	2	2	15†
6	D minor	3	3	8
		2	6	11†
		2	6	12†
		14	12†	11
		3	2	7†
		4	10	6
12	F minor	3	15	4†
13	F♯ major	3	2	8
14	F♯ minor	7	18	5†
15	G major	2	2	15
16	G minor	3	3	4
17	A♭ major	3	2	7†
18	G♯ minor	5	5	5
19	A major	2	4	7
20	A minor	5	5	5
21	B♭ major	4	4	14
22	B♭ minor	2	3	6†
23	B major	2	11	11
24	B minor	3	3	7
	Average	[illegible]	5.25 (4.79)	8.71 (8.21)

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-60

Key-Finding: Comparisons

J.S. Bach's *Well-Tempered Clavier* Bk 1

LISTEN

Fugue No. 5 in D major

	Book 1	Steps to key		
	Fugue subjects	CEG[a]	PTPM	SMA
1	C major	2	2	16†[b]
2	C minor	5	5	5
3	C♯ major	6	7	16
4	C♯ minor	3	3	4
5	D major	2	2	15†
6	D minor	3	3	8
7	E♭ major	2	6	11†
		2	6	12†
		14	12†	11
		3	2	7†
		4	10	6
12	F minor	3	15	4†
13	F♯ major	3	2	8
14	F♯ minor	7	18	5†
15	G major	2	2	15
16	G minor	3	3	4
17	A♭ major	3	2	7†
18	G♯ minor	5	5	5
19	A major	2	4	7
20	A minor	5	5	5
21	B♭ major	4	4	14
22	B♭ minor	2	3	6†
23	B major	2	11	11
24	B minor	3	3	7
	Average	3.75 (3.57)	5.25 (4.79)	8.71 (8.21)

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-61

Key-Finding: Comparisons

J.S. Bach's *Well-Tempered Clavier* Bk 1

LISTEN

Fugue No. 15 in G major

	Book 1	Steps to key		
	Fugue subjects	CEG[a]	PTPM	SMA
1	C major	2	2	16†[b]
2	C minor	5	5	5
3	C♯ major	6	7	16
4	C♯ minor	3	3	4
5	D major	2	2	15†
6	D minor	3	3	8
7	E♭ major	2	6	11†
8	D♯ minor	2	6	12†
		14	12†	11
		3	2	7†
		4	10	6
		3	15	4†
		3	2	8
14	F♯ minor	7	18	5†
15	G major	2	2	15
16	G minor	3	3	4
17	A♭ major	3	2	7†
18	G♯ minor	5	5	5
19	A major	2	4	7
20	A minor	5	5	5
21	B♭ major	4	4	14
22	B♭ minor	2	3	6†
23	B major	2	11	11
24	B minor	3	3	7
	Average	3.75 (3.57)	5.25 (4.79)	8.71 (8.21)

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-62

Key-Finding: Comparisons

J.S. Bach's *Well-Tempered Clavier* Bk 1

LISTEN

Fugue No. 21 in B♭ major

	Book 1	Steps to key		
	Fugue subjects	CEG[a]	PTPM	SMA
1	C major	2	2	16†[b]
2	C minor	5	5	5
3	C♯ major	6	7	16
4	C♯ minor	3	3	4
5	D major	2	2	15†
6	D minor	3	3	8
7	E♭ major	2	6	11†
		2	6	12†
		14	12†	11
		3	2	7†
		4	10	6
		3	15	4†
13	F♯ major	3	2	8
14	F♯ minor	7	18	5†
15	G major	2	2	15
16	G minor	3	3	4
17	A♭ major	3	2	7†
18	G♯ minor	5	5	5
19	A major	2	4	7
20	A minor	5	5	5
21	B♭ major	4	4	14
22	B♭ minor	2	3	6†
23	B major	2	11	11
24	B minor	3	3	7
	Average	3.75 (3.57)	5.25 (4.79)	8.71 (8.21)

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-63

Key-Finding: Comparisons

J.S. Bach's *Well-Tempered Clavier* Bk 1

LISTEN

Fugue No. 14 in F♯ minor

	Book 1	Steps to key		
	Fugue subjects	CEG[a]	PTPM	SMA
1	C major	2	2	16†[b]
2	C minor	5	5	5
3	C♯ major	6	7	16
4	C♯ minor	3	3	4
5	D major	2	2	15†
6	D minor	3	3	8
		2	6	11†
		2	6	12†
		14	12†	11
		3	2	7†
		4	10	6
12	F minor	3	15	4†
13	F♯ major	3	2	8
14	F♯ minor	7	18	5†
15	G major	2	2	15
16	G minor	3	3	4
17	A♭ major	3	2	7†
18	G♯ minor	5	5	5
19	A major	2	4	7
20	A minor	5	5	5
21	B♭ major	4	4	14
22	B♭ minor	2	3	6†
23	B major	2	11	11
24	B minor	3	3	7
	Average	3.75 (3.57)	5.25 (4.79)	8.71 (8.21)

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-64

Key-Finding: Comparisons

J.S. Bach's *Well-Tempered Clavier* Bk 1

LISTEN

Fugue No. 9 in E major

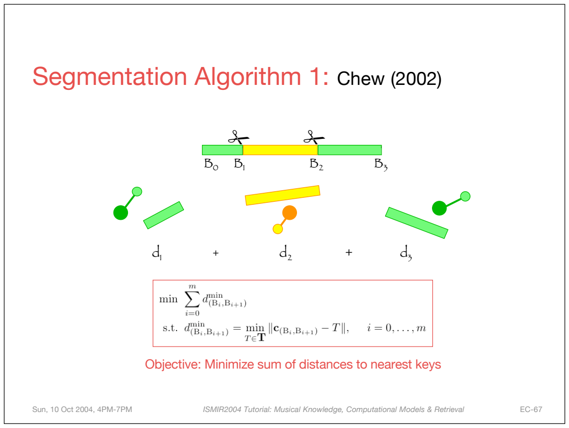
	Book 1	Steps to key		
	Fugue subjects	CEG[a]	PTPM	SMA
1	C major	2	2	16†[b]
2	C minor	5	5	5
3	C♯ major	6	7	16
4	C♯ minor	3	3	4
5	D major	2	2	15†
6	D minor	3	3	8
7	E♭ major	2	6	11†
8	D♯ minor	2	6	12†
9	E major	14	12†	11
10	E minor	3	2	7†
11	F major	4	10	6
12	F minor	3	15	4†
13	F♯ major	3	2	8
14	F♯ minor	7	18	5†
		2	2	15
		3	3	4
		3	2	7†
		5	5	5
		2	4	7
20	A minor	5	5	5
21	B♭ major	4	4	14
22	B♭ minor	2	3	6†
23	B major	2	11	11
24	B minor	3	3	7
	Average	3.75 (3.57)	5.25 (4.79)	8.71 (8.21)

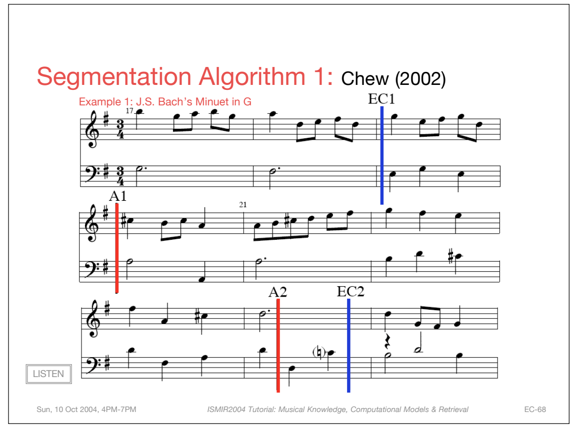
Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-65

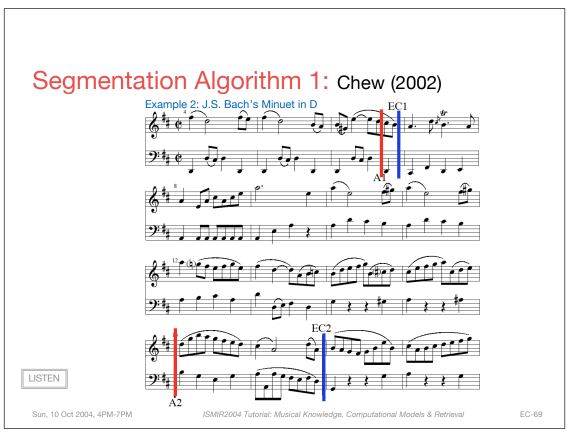
Segmentation

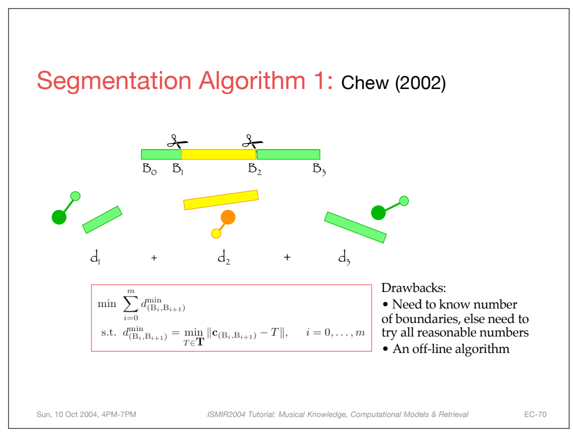
- Extensions of the CEG algorithm
  - Segmentation Algorithm 1 (Chew 2002)
  - Segmentation Algorithm 2: Argus (Chew 2004)
- Extension of Krumhansl & Schmuckler
  - Dynamic programming approach (Temperley 1999)

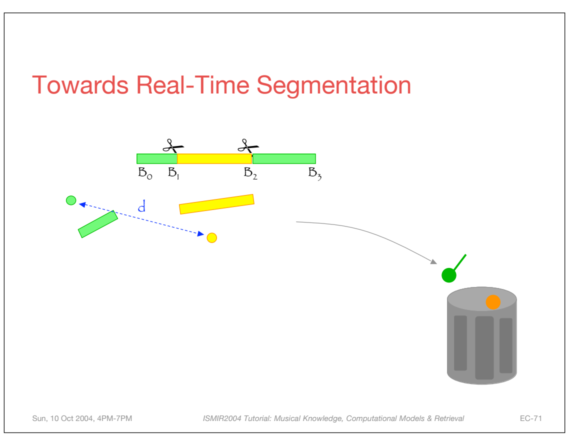
Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-66

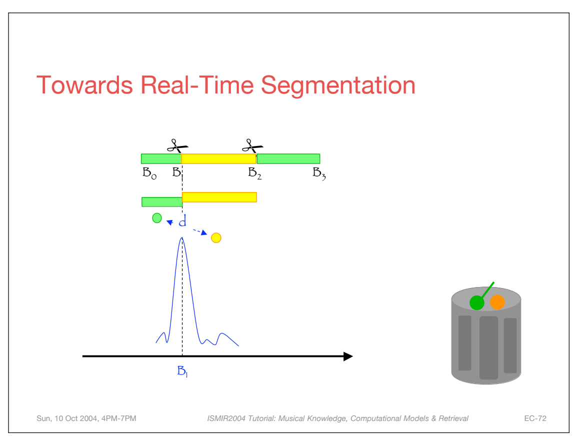

Segmentation Algorithm 1: Chew (2002)
B0 B1 B2 B3
d1 + d2 + d3
$\min \sum_{i=0}^{m} d^{\min}_{(B_i,B_{i+1})}$
s.t. $d^{\min}_{(B_i,B_{i+1})} = \min_{T \in \mathbf{T}} \|\mathbf{c}_{(B_i,B_{i+1})} - T\|, \quad i = 0, \ldots, m$
Objective: Minimize sum of distances to nearest keys
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-67


Segmentation Algorithm 1: Chew (2002)
Example 1: J.S. Bach's Minuet in G
EC1
A1
A2
EC2
LISTEN
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-68


Segmentation Algorithm 1: Chew (2002)
Example 2: J.S. Bach's Minuet in D
EC1
A1
EC2
A2
LISTEN
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-69

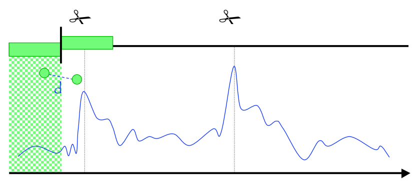

Segmentation Algorithm 1: Chew (2002)
B0 B1 B2 B3
d1 + d2 + d3
$\min \sum_{i=0}^{m} d^{\min}_{(B_i,B_{i+1})}$
s.t. $d^{\min}_{(B_i,B_{i+1})} = \min_{T \in \mathbf{T}} \|\mathbf{c}_{(B_i,B_{i+1})} - T\|, \quad i = 0, \ldots, m$
Drawbacks:
• Need to know number of boundaries, else need to try all reasonable numbers
• An off-line algorithm
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-70


Towards Real-Time Segmentation
B0 B1 B2 B3
d
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-71


Towards Real-Time Segmentation
B0 B1 B2 B3
d
B1
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-72

Segmentation Algorithm 2: Argus (Chew 2004)

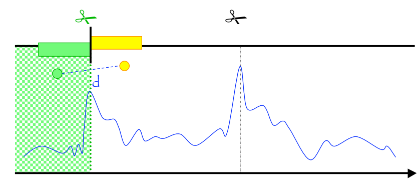
Real-time segmentation algorithm …



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-73

Segmentation Algorithm 2: Argus (Chew 2004)

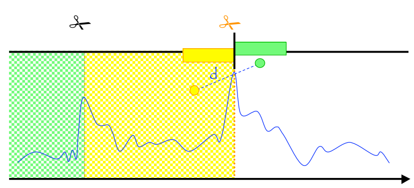
Real-time segmentation algorithm …



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-74

Segmentation Algorithm 2: Argus (Chew 2004)

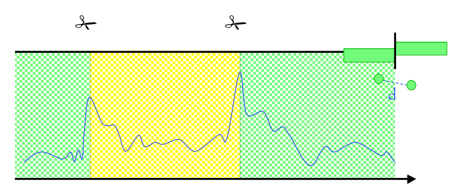
Real-time segmentation algorithm …



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-75

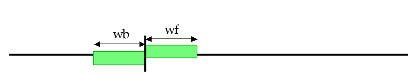
Segmentation Algorithm 2: Argus (Chew 2004a)

Real-time segmentation algorithm …



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-76

Segmentation Algorithm 2: Argus (Chew 2004a)

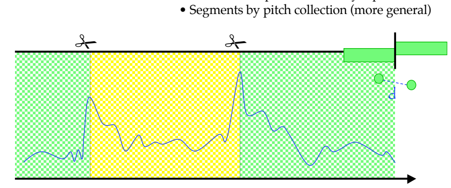


Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-77

Segmentation Algorithm 2: Argus (Chew 2004a)

Real-time segmentation algorithm …

Advantages:
- Computes in real-time, $O(n)$
- Eliminates dependence on key representations
- Segments by pitch collection (more general)



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-78

Segmentation Algorithm 2: Argus (Chew 2004b)

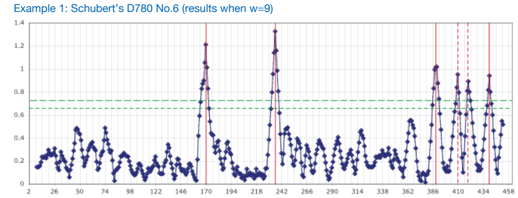
Example 1: Schubert's D780 No.6

LISTEN

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-79

Segmentation Algorithm 2: Argus (Chew 2004b)

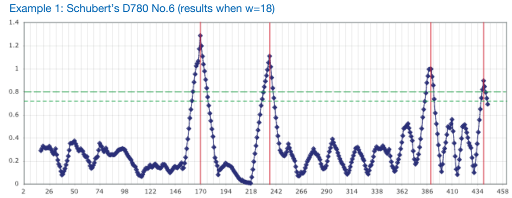
Example 1: Schubert's D780 No.6 (results when w=9)



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-80

Segmentation Algorithm 2: Argus (Chew 2004b)

Example 1: Schubert's D780 No.6 (results when w=18)



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-81

Segmentation Algorithm 2: Argus (Chew 2004b)

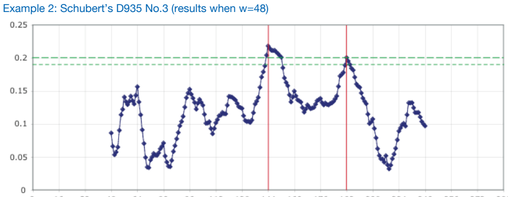
Example 2: Schubert's D935 No.3

LISTEN

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-82

Segmentation Algorithm 2: Argus (Chew 2004b)

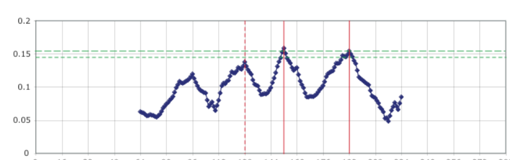
Example 2: Schubert's D935 No.3 (results when w=48)



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-83

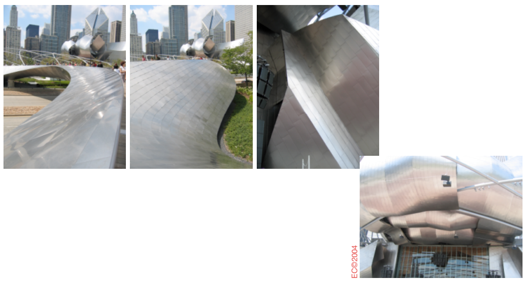
Segmentation Algorithm 2: Argus (Chew 2004b)

Example 2: Schubert's D935 No.3 (results when w=64)



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-84

Gehry's Pritzker Pavillion and BP bridge, Chicago



Sun, 10 Oct 2004, 4PM-7PM *ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval* EC-85

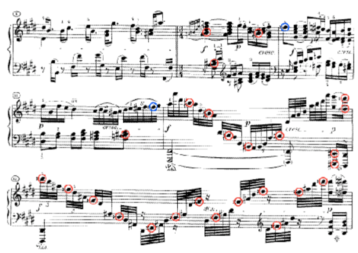
Goals for the afternoon

- Music Perception and Cognition
  - What can we hear?
  - How can we describe it?
- Modeling Musical Intelligence
  - Focus on pitch structures
  - [1] Modeling tonality
  - Computational music cognition
    - [2] Key Finding
    - [3] Segmentation
    - [4] Pitch Spelling

Sun, 10 Oct 2004, 4PM-7PM *ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval* EC-86

Transcription example

Beethoven *Piano Sonata Op.109*



LISTEN

[Finale]

Sun, 10 Oct 2004, 4PM-7PM *ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval* EC-87

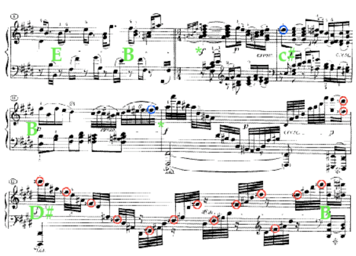
Transcription example

Beethoven *Piano Sonata Op.109*

[Sibelius]

Sun, 10 Oct 2004, 4PM-7PM *ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval* EC-88

Why is the example hard?



LISTEN

Sun, 10 Oct 2004, 4PM-7PM *ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval* EC-89

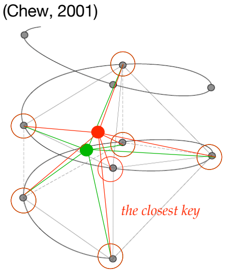
Pitch Spelling

- What is spelling? Why spell?
- Three algorithms:
  - Cumulative *c.e.* (Chew & Chen 2003a)
  - Sliding window *c.e.* (Chew & Chen 2003b)
  - Bootstrapping (Chew & Chen 2003b, (in press))
- Joint work with Yun-Ching Chen

Sun, 10 Oct 2004, 4PM-7PM *ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval* EC-90

Recall: Key-Finding

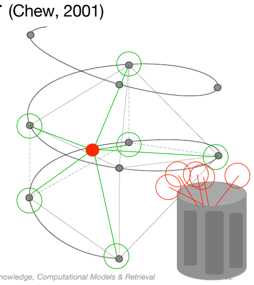
- Center of Effect Generator (Chew, 2001)
  - clustering of pitches in a key
  - generate center of effect
  - perform nearest neighbor search for closest key



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-91

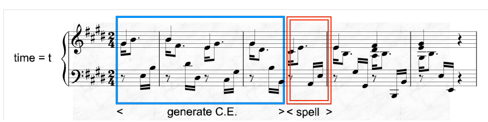
Recall: Key-Finding

- Center of Effect Generator (Chew, 2001)
  - clustering of pitches in a key
  - generate center of effect
  - perform nearest neighbor search for closest key



Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval

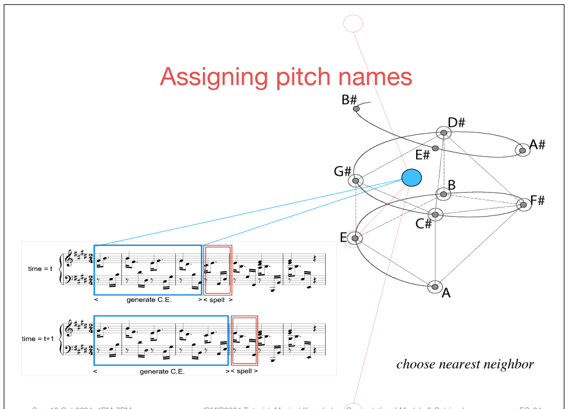
Algorithm 1: cumulative *c.e.*



LISTEN

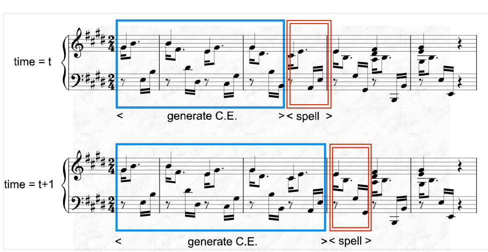
Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-93

Assigning pitch names



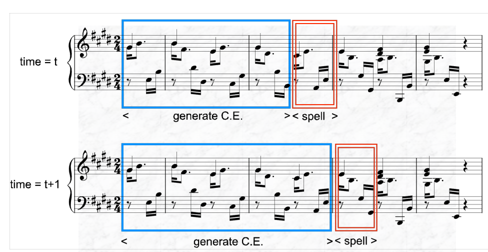
Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-94

Algorithm 1: cumulative *c.e.*



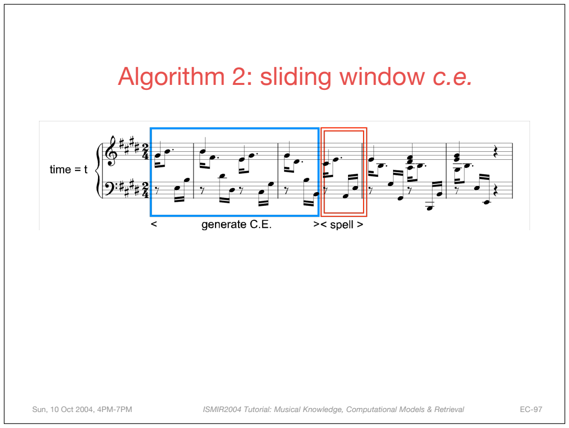
Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-95

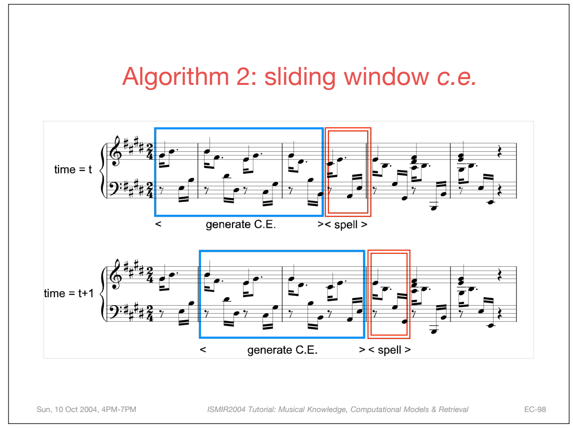
Algorithm 1: cumulative *c.e.*

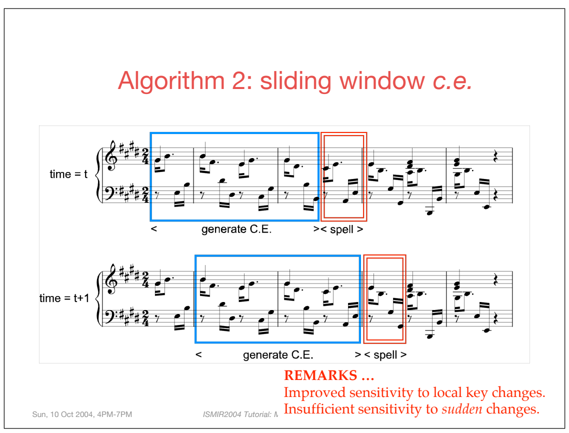


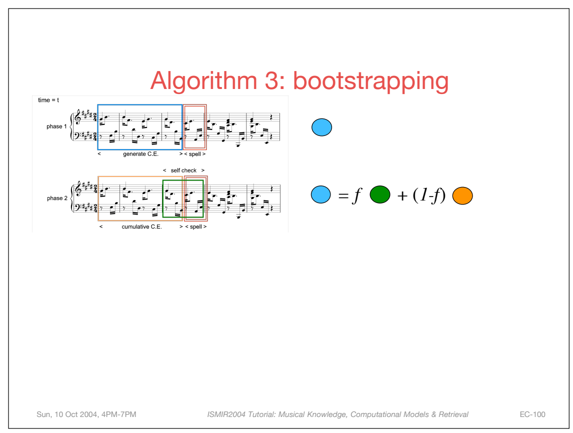
REMARKS …
Insufficient sensitivity to key change.
No knowledge of voice-leading conventions

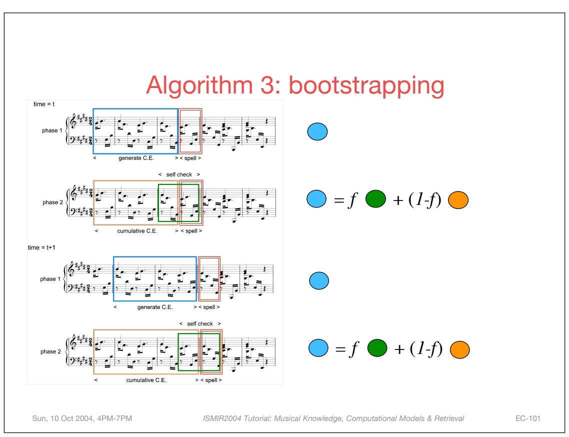
Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: M

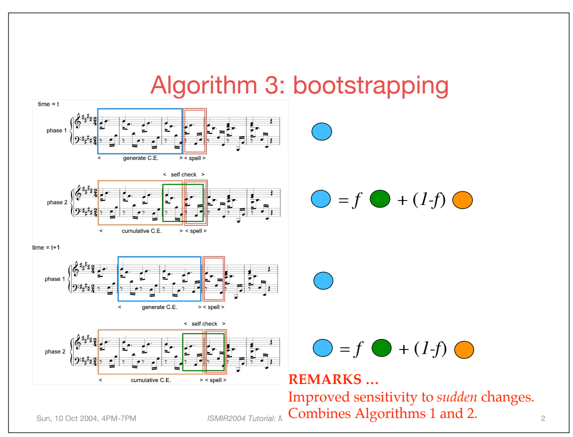

Algorithm 2: sliding window *c.e.*
time = t
< generate C.E. >< spell >
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-97


Algorithm 2: sliding window *c.e.*
time = t
< generate C.E. >< spell >
time = t+1
< generate C.E. >< spell >
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-98


Algorithm 2: sliding window *c.e.*
time = t
< generate C.E. >< spell >
time = t+1
< generate C.E. >< spell >
REMARKS ...
Improved sensitivity to local key changes.
Insufficient sensitivity to *sudden* changes.
Sun, 10 Oct 2004, 4PM-7PM


Algorithm 3: bootstrapping
time = t
phase 1
< generate C.E. >< spell >
< self check >
phase 2
< cumulative C.E. >< spell >
= *f* + (*1-f*)
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-100


Algorithm 3: bootstrapping
time = t
phase 1
phase 2
time = t+1
phase 1
phase 2
= *f* + (*1-f*)
= *f* + (*1-f*)
Sun, 10 Oct 2004, 4PM-7PM
ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval
EC-101


Algorithm 3: bootstrapping
time = t
phase 1
phase 2
time = t+1
phase 1
phase 2
= *f* + (*1-f*)
= *f* + (*1-f*)
REMARKS ...
Improved sensitivity to *sudden* changes.
Combines Algorithms 1 and 2.
Sun, 10 Oct 2004, 4PM-7PM

Computational Results

Method	Parameters	Errors	Percentage correct
Beethoven Op.109 (1st movement) , 1516 notes			
Cumulative		73	95.18
Sliding Window	w = 4	31	98.00
	w = 8	47	96.90
Bootstrapping	w=4, r = 2, f = 0.6	28	98.15
	w=4, r = 3, f = 0.8	27	98.22
	w=8, r = 2, f = 0.9	31	97.96
	w=8, r = 6, f = 0.9	27	98.22
Beethoven Op.79 (3rd movement) , 1375 notes			
Cumulative		1	99.93
Sliding Window	w = 4	1	99.93

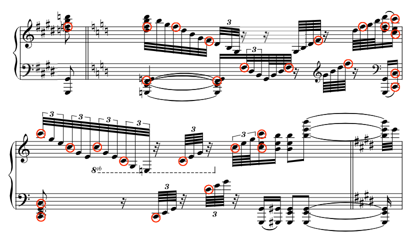
Op.79 Movement 3: one error

LISTEN

Computational Results

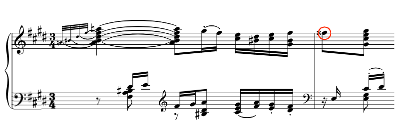
Method	Parameters	Errors	Percentage correct
Beethoven Op.109 (1st movement) , 1516 notes			
Cumulative		73	95.18
Sliding Window	w = 4	31	98.00
	w = 8	47	96.90
Bootstrapping	w=4, r = 2, f = 0.6	28	98.15
	w=4, r = 3, f = 0.8	27	98.22
	w=8, r = 2, f = 0.9	31	97.96
	w=8, r = 6, f = 0.9	27	98.22
Beethoven Op.79 (3rd movement) , 1375 notes			
Cumulative		1	99.93
Sliding Window	w = 4	1	99.93

Insensitivity to key change



LISTEN

No knowledge of voice leading



LISTEN

Computational Results

Method	Parameters	Errors	Percentage correct
Beethoven Op.109 (1st movement) , 1516 notes			
Cumulative		73	95.18
Sliding Window	w = 4	31	98.00
	w = 8	47	96.90
Bootstrapping	w=4, r = 2, f = 0.6	28	98.15
	w=4, r = 3, f = 0.8	27	98.22
	w=8, r = 2, f = 0.9	31	97.96
	w=8, r = 6, f = 0.9	27	98.22
Beethoven Op.79 (3rd movement) , 1375 notes			
Cumulative		1	99.93
Sliding Window	w = 4	1	99.93

Computational Results

Method	Parameters	Errors	Percentage correct
Beethoven Op.109 (1st movement), 1516 notes			
Cumulative		73	95.18
Sliding Window	w = 4	31	98.00
	w = 8	47	96.90
Bootstrapping	w=4, r = 2, f = 0.6	28	98.15
	w=4, r = 3, f = 0.8	27	98.22
	w=8, r = 2, f = 0.9	31	97.96
	w=8, r = 6, f = 0.9	27	98.22
Beethoven Op.79 (3rd movement), 1375 notes			
Cumulative		1	99.93
Sliding Window	w = 4	1	99.93

Transcription example

Beethoven *Piano Sonata Op.109*

LISTEN

[Spiral Array Pitch Spelling (blue)]

Related Work 2

Meredith's 2004 comparison of pitch spelling algorithms.

Author(s)	*Method*	*Percentage correct*			
		Vivaldi 1678-1741	*Bach* 1685-1750	*Mozart* 1756-1791	*Beethoven* 1770-1827
		223678 notes	627083 notes	172097 notes	48659 notes
Meredith	*ps13*	99.23	99.37	98.49	98.54
Cambouropoulos	*Interval Opt*	99.13	97.92	98.58	98.63
Longuet-Higgins	*Line-of-Fifths*	98.48	98.49	96.26	97.46
Temperley	*Line-of-Fifths*	98.69	99.74	93.40	92.34
AVERAGE		98.88	98.88	96.68	96.74

References-1

Chew (2000). **Towards a Mathematical Model of Tonality**. MIT PhD dissertation.
Chew (2001). "Modeling Tonality: Applications to Music Cognition". *Proc CogSci2001*, 206-211.
Chew (2002). "The Spiral Array: An Algorithm to Determine Key Boundaries". ICMAI. Springer LNCS/LNAI #2445, 18-31.
Chew (2003). "Thinking Out of the Grid and Inside the Spiral". USC IMSC Technical Report.
Chew (2004). Class syllabi and notes for ISE599 Engineering Approaches to Music Perception and Cognition. *http://www-classes.usc.edu/engr/ise/599muscog/2004*
Chew (2004a). "Regards on Two Regards by Messiaen: Automatic Segmentation Using the Spiral Array". *Proc SMC'04.*
Chew (2004b). "Slicing It All Ways: Mathematical Models for Tonal Induction, Approximation and Segmentation Using the Spiral Array". USC IMSC Technical Report. To appear (after revisions) in INFORMS Journal on Computing.
Chew & Chen (2003a). "Mapping MIDI to the Spiral Array: Disambiguating Pitch Spellings." *Proc ICS*. Kluwer.
Chew & Chen (2003b). "Determining Context-Defining Windows: Pitch Spelling using the Spiral Array". ISMIR 2003.
Chew & Chen (in press). "Pitch Spelling using the Spiral Array". Computer Music Journal.

References-2

Cohn (1997). "Neo -Riemannian Operations, Parsimonious Trichords, and their Tonnetz Representations." J. of Music Theory 41/1: 1-66
Cohn (1998). "Introduction to Neo-Riemannian Theory: A Survey and a Historical Perspective". J. of Music Theory 42:167–180.
Krumhansl (1990). **Cognitive Foundations of Musical Pitch**. Oxford U. Press.
Lerdahl (2001). **Tonal Pitch Space**. Oxford U. Press.
Lewin (1987). **Generalized Musical Intervals and Transformations**. Yale U. Press.
Longuet-Higgins (1962a). "Letter to a Musical Friend". Music Review 23, 244-248.
Longuet-Higgins (1962b). "Second Letter to a Musical Friend". Music Review 23, 271-280.
Longuet-Higgins & Steedman (1971). "On Interpreting Bach". Machine Intelligence 6, 221-241.
Meredith (2003). "Pitch Spelling Algorithms". *Proc ESCOM*.
Meredith (2004). "Comparing Pitch Spelling Algorithms on a Large Corpus of Music". CMMR. Spring LNCS #2771.
Shepard (1982). "Structural Representations of Musical Pitch". The Psychology of Music 11, 343-390.
Temperley (1999). "What's Key for Key? The Krumhansl-Schmuckler Key-Finding Algorithm Reconsidered". Music Perception 17 #1, 65-100.
Temperley (2002). **The Cognition of Basic Musical Structures**. MIT Press.

The two visual themes

- Theme I: Chew's spiral array
  - Modeling tonal relations
  - Determining pitch context (key)
  - Modulation and segmentation
  - Naming things in context: pitch spelling
- Theme II: Gehry's metallic objects
  - Walt Disney Hall, Los Angeles
  - Pritzker Hall and bridge, Chicago

NEXT: Gehry's Guggenheim Museum, Bilbao

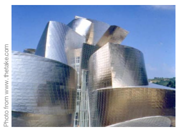


Photo from www.thetake.com

Sun, 10 Oct 2004, 4PM-7PM ISMIR2004 Tutorial: Musical Knowledge, Computational Models & Retrieval EC-115